

Nucleon Form Factors in a Light-Cone Quark Model with Two-Photon Exchange

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Abstract

We estimate the two-photon exchange corrections to both proton and neutron electromagnetic physical observables in a relativistic light cone quark model. At a fixed Q^2 the corrections are found to be small in magnitudes, but strongly dependent on the scattering angle. Our results are comparable to those obtained from simple hadronic model in a medium momentum transfer region.

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1 Introduction

In the experimental point of view, electron-nucleon scattering is one of time-honored tools to access the information of the intrinsic structures of the nucleon. These structures are partly reflected by the Sachs electric ($G_E(Q^2)$) and magnetic ($G_M(Q^2)$) form factors. So far there are two experimental techniques to

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detect these form factors or the form factors ratio $R = \mu_p G_E / G_M$. The traditional one is the Rosenbluth separation method [1], which extracts the form factors ratio from the angular dependence of the elastic electron-proton scattering cross section. In the one-photon exchange approximation, the differential cross section for the $e N$ elastic scattering process is:

$$\frac{d\sigma}{d\Omega} \propto G_M^2(Q^2) + \frac{\epsilon}{\tau} G_E^2(Q^2), \quad (1)$$

where $Q^2 = -q^2$ is the momentum transfer squared, the dynamics factor $\tau = Q^2/4M^2$ and the photon polarization parameter ϵ relates to the laboratory scattering angle θ by $\epsilon = (1 + 2(1 + \tau) \tan^2 \theta/2)^{-1}$. For a given value of Q^2 , Eq. (1) shows that it is sufficient to determine the form factors by measuring the differential cross sections at two different values of ϵ .

Polarized lepton beams provide another way to access the form factors [2]. In the one-photon exchange approximation, the polarization of the recoiling proton along its motion (P_z) is proportional to G_M^2 while the component perpendicular to the motion (P_x) is proportional to $G_E G_M$. It is much easier to measure the ratios of polarizations. This method has been used mainly to determine the electromagnetic form factors ratio R through a measurement of P_x/P_z using [3]

$$\frac{P_x}{P_z} = -\sqrt{\frac{2\epsilon}{\tau(1+\epsilon)}} \frac{G_E}{G_M}. \quad (2)$$

In the framework of one-photon exchange approximation, one, therefore, has two independent measurements of the form factors ratio R . Recently, this ratio has been observed at the Jefferson Laboratory by the polarized method [4–6]. It came as a surprise that the newly measured form factors ratio is much different from the results of the Rosenbluth separations [7–9]. As shown in Eq. (1), the form factors extracted from the Rosenbluth separation method are strongly lie on ϵ –dependent corrections at large Q^2 region. After re-analyzing the next leading order QED corrections, one finds that the two-photon exchange (TPE) process should be restudied.

In previous literatures, there have been two different methods to study the TPE contributions in the electron proton scattering process. One is the simple hadronic model, in which the intermediate states of the TPE process are taken as baryons. The known MT corrections [10, 11], which have been included

in the Rosenbluth separation method, are based on this model. It should be emphasized that in the MT corrections, the loop integrals of TPE contributions are evaluated by setting one of the photon's momenta to be zero in both numerators and denominators of the amplitudes, while the rest parts are ignored. In Ref. [12], different from the MT corrections, the TPE contributions are considered by neglecting one of the photon's momenta only in the numerators of the amplitudes. Furthermore, in Ref. [13], the TPE contributions are evaluated by keeping the full numerators with nucleon intermediate state. The newly estimated results show that the corrections can at least partly reconcile the apparent discrepancy between the two separation methods. In the further study more intermediate states have been taken into considerations [14, 15].

Another approach to deal with the TPE process is the quark model. In this model, the TPE contributions are firstly considered in the quark level and then extended from the quark level to the baryon level. In Ref. [16, 17], the contributions of TPE process have been studied at large momentum transfer and wide scattering angle region in a parton model. In these cases, the quark mass can be neglected. The parton model can work effectively with large momentum transfer. However, the contributions of the TPE process are still unknown in the medium Q^2 region. In this work, we try to calculate the TPE contributions in a light-cone quark model and compare our results with the predictions of the simple hadronic model. The contributions of the TPE to unpolarized differential cross sections and polarized observables will be evaluated at medium Q^2 and small ϵ regions.

This paper is organized as follows. In section 2, we re-study the TPE process in quark level with massive quarks and show a simple comparison of our results with those in Ref. [16, 17]. In section 3, a brief introduction of the light-cone constituent quark model will be addressed, and then we give our analytical expressions of the TPE contributions in this model. The numerical results and discussions about the TPE corrections to the differential cross sections and to the polarized observables are displayed in section 4.

2 Two-Photon Exchange Process in Quark Level

As the first step, The TPE contributions in quark level (As shown in Fig. 1) will be estimated. According to parity, time-reversal and lepton helicity conservation, the amplitude of the TPE process in the quark level can be expanded in terms of three independent Lorentz structures. Generally, the amplitude can be expressed as,

$$\mathcal{M}_{eq}^{2\gamma} = -i \frac{(e_q e)^2}{q^2} \bar{u}(k') \gamma_\mu u(k) \bar{u}(p'_q) \left(\tilde{f}_1 \gamma^\mu + i \tilde{f}_2 \frac{\sigma^{\mu\nu} q_\nu}{2m_q} + \tilde{f}_3 \frac{\gamma \cdot K P_q^\mu}{m_q^2} \right) u(p_q), \quad (3)$$

with $P_q = (p_q + p'_q)/2$ and $K = (k + k')/2$. Here \tilde{f}_i , $\{i = 1, 2, 3\}$ are the functions of Mandelstam variables s', u', t' in quark level and

$$s' = (k + p_q)^2, \quad u' = (k - p'_q)^2, \quad t' = (p'_q - p_q)^2 = (k - k')^2. \quad (4)$$

If we ignore the electron mass, the Mandelstam variables satisfy

$$s' + u' + t' = 2m_q^2. \quad (5)$$

In the actual calculations, the amplitudes of TPE processes corresponding to the Feynman diagrams in Fig. 1 are:

$$\begin{aligned} \mathcal{M}_{eq(a)}^{2\gamma} &= (ee_q)^2 \int \frac{d^4 \ell}{(2\pi)^4} \frac{\bar{u}(k') \gamma^\mu \hat{\ell} \gamma^\nu u(k)}{[\ell^2 - m_e^2][(k - \ell)^2 - \lambda^2]} \\ &\quad \times \frac{\bar{u}(p'_q) \gamma_\mu (\hat{p}_q + \hat{k} - \hat{\ell} + m_q) \gamma_\nu u(p_q)}{[(\ell - k')^2 - \lambda^2][(p_q + k - \ell)^2 - m_q^2]}, \\ \mathcal{M}_{eq(b)}^{2\gamma} &= (ee_q)^2 \int \frac{d^4 \ell}{(2\pi)^4} \frac{\bar{u}(k') \gamma^\mu \hat{\ell} \gamma^\nu u(k)}{[\ell^2 - m_e^2][(k - \ell)^2 - \lambda^2]} \\ &\quad \times \frac{\bar{u}(p'_q) \gamma_\nu (\hat{p}_q - \hat{k}' + \hat{\ell} + m_q) \gamma_\mu u(p_q)}{[(\ell - k')^2 - \lambda^2][(p_q - k' + \ell)^2 - m_q^2]}, \end{aligned} \quad (6)$$

where $\hat{k} \equiv \gamma \cdot k$. The factors $\tilde{f}_i, \{i = 1, 2, 3\}$ in Eq. (3), therefore, can be extracted from the sum of above two amplitudes. In Fig. 2, we give a comparison of our results (quark mass $m_q = 0.22 \text{ GeV}$) with those in Ref. [16, 17] in the unit of percent. In the figure the charge of the quark is assumed as $e_q = e$. At $Q^2 = 6 \text{ GeV}^2$, one sees that the real parts of our results of \tilde{f}_1 and \tilde{f}_3 are close to the results with massless quark, especially at large ϵ_q region. Our result about \tilde{f}_2 with massive quark is comparable to \tilde{f}_1

and \tilde{f}_3 and therefore, it should not be ignored in our calculations. Further more, at $Q^2 = 0.5 \text{ GeV}^2$ the discrepancy of our results with those in Ref. [16,17] are even larger. Thus, one can conclude that at large Q^2 region massless quark may be a good approximation, however, at medium Q^2 region the quark mass is un-neglectable. These conclusions are also suitable for the case imaginary parts of $\tilde{f}_i, \{i = 1, 2, 3\}$.

For further use, we separate the amplitude \tilde{f}_1 into soft and hard parts, i.e. $\tilde{f}_1 = \tilde{f}_1^{soft} + \tilde{f}_1^{hard}$. The soft part can be obtained from Eq. (6) by neglecting one of photon momenta in the numerators of the amplitudes. Then the soft part of \tilde{f}_1 is,

$$\tilde{f}_1^{soft} = -\frac{\alpha}{\pi} \ln \left| \frac{s' - m_q^2}{s' + t' - m_q^2} \right| \ln \left| \frac{t'}{\lambda^2} \right|. \quad (7)$$

One sees \tilde{f}_1^{soft} , which is proportional to $\ln \lambda^2$, is IR divergent. The hard part of \tilde{f}_1 and other structure amplitudes \tilde{f}_2, \tilde{f}_3 are IR finite.

Here we must notice that in the parton model [16,17], the soft part of \tilde{f}_1 is separated out by replacing one of the photon's momenta by zero in both numerators and denominators of the amplitudes, then one can get a three-point Passarino-Veltman function [18], which has no analytical representation and is much more complicate for massive quark.

3 Two-Photon Exchange in Light-Cone Constituent Quark Model

The second step of studying the TPE contributions in quark model is to embed the amplitudes of quark level to baryon level. In parton model [16,17], the general parton distribution functions are employed. Here we perform similar calculations in a light cone quark model.

It is well-known that the constituent quark model (CQM) developed within a light cone framework [19–22] appears to be an interesting tool for investigating the electromagnetic properties of hadrons. For relativistic bound states it provides a momentum-space Fock-state basis defined at $t^+ = t + z$ on the light cone, rather than the more conventional equal-time wave functions of the instant form. On the light cone, it is consistent to take particles on their mass shell in general. This feature allows using light-cone

spinors for quarks in multi-quark hadron wave functions rather than propagators in the instant form.

In the light-cone quark model, for a three-quark system, the configuration is conveniently described in terms of the longitudinal-momentum fractions (Bjorken-Feynman variables) and relative momentum variables:

$$\begin{aligned} x_j &= \frac{p_j^+}{p^+}, \quad \sum_{j=1}^3 x_j = 1, \quad 0 \leq x_j \leq 1, \\ q_3 &= \frac{x_2 p_1 - x_1 p_2}{x_1 + x_2}, \\ Q_3 &= (x_1 + x_2)p_3 - x_3(p_1 + p_2) = p_3 - x_3 p_3. \end{aligned} \quad (8)$$

Where p and p_i , $\{i = 1, 2, 3\}$ are the momenta of nucleon and the quarks. Here $p^+ = \sum_{j=1}^3 p_j^+$ reflects the conservation of the total momentum p . The crucial properties of the relative momentum variables are $Q_3^+ = q_3^+ = 0$, therefore they are space-like four vectors $q_3 = -\mathbf{q}_{3\perp}^2$, $Q_3^2 = -\mathbf{Q}_{3\perp}^2$. These six relative variables $x_1, x_2, \mathbf{q}_{3\perp}, \mathbf{Q}_{3\perp}$ are translational invariant and invariant under the three light-cone boost [23].

In present work, the calculations are performed in a symmetric frame [24, 25], which are the same as those in the parton model [16, 17]. In such a frame the Mandelstam variables in baryon level are

$$\begin{aligned} s &= (p + k)^2 = -\frac{1 + \eta}{4\eta}t + (1 + \eta)M^2, \\ u &= (p - k')^2 = \frac{1 - \eta}{4\eta}t + (1 - \eta)M^2, \\ t &= (k - k')^2 = (p' - p)^2, \end{aligned} \quad (9)$$

with $\eta = (s - u - 2\sqrt{M^4 - su})/(4M^2 - t)$. Moreover, in above frame, we have a large p^+ (the $' +'$ component of the initial proton), then the transverse momenta of the spectator quarks are supposed to be small relative to p^+ and can be neglected. Based on such approximation, the Mandelstam variables of the quark level can directly connect to those of the baryon level by

$$\begin{aligned} s' &= -\frac{(x + \eta)^2}{4x\eta}t + \frac{x + \eta}{x}m_q^2, \\ u' &= \frac{(x - \eta)^2}{4x\eta}t + \frac{x - \eta}{x}m_q^2, \end{aligned} \quad (10)$$

where $x = p_q^+/p^+$ is the ratio of $' +'$ component of the active quark (quark interacting with the external field) momentum and nucleon momentum.

The symmetric frame can be taken as a special Drell-Yan frame with the essential feature $q^+ = 0$ [20, 26]. In such a frame, the form factors F_1 and F_2 under one-photon exchange approximation can be determined from the J^+ matrix elements alone, i.e.,

$$\begin{aligned} eF_1(q^2) &= \frac{M}{P^+} \langle N(P') \uparrow | J^+ | N(P) \uparrow \rangle, \\ \frac{q_L}{2M} eF_2(q^2) &= -\frac{M}{P^+} \langle N(P') \uparrow | J^+ | N(P) \downarrow \rangle. \end{aligned} \quad (11)$$

The matrix element for the three-quark nucleon wave function ψ_N reads,

$$\langle N\lambda' | \frac{J^+}{P^+} | N\lambda \rangle = \sum_{j=1}^3 \int d\Gamma \psi_N^\dagger(x'_i, q'_3, Q'_3, \lambda') \frac{J_q^+}{\sqrt{p_j'^+ p_j^+}} \psi_N(x_i, q_3, Q_3, \lambda), \quad (12)$$

with the invariant phase-space volume element

$$d\Gamma = \frac{1}{(2\pi)^6} d^2\mathbf{q}_{3\perp} d^2\mathbf{Q}_{3\perp} \delta\left(\sum_{i=1}^3 x_i - 1\right) \prod_{i=1}^3 \frac{dx_i}{x_i}, \quad (13)$$

and λ denotes the spin of nucleon. J_q^μ is the electromagnetic current of the active quark with charge e_j .

In the one-photon approximation, we have,

$$J_{q\mu}^{1\gamma} = e_j \bar{u}(p'_j) \gamma^\mu u(p_j) \quad (14)$$

After considering TPE contributions, the electromagnetic current of the active quark can be derived from Eq. (3), it is,

$$J_{q\mu}^{2\gamma} = e(e_j)^2 \bar{u}(p'_j) \left(\tilde{f}_1 \gamma^\mu + i \tilde{f}_2 \frac{\sigma^{\mu\nu} q_\nu}{2m_q} + \tilde{f}_3 \frac{\gamma \cdot K P_q^\mu}{m_q^2} \right) u(p_j). \quad (15)$$

Meanwhile, in the baryon level, after including the TPE contributions, a new term will be introduced in the nucleon electromagnetic vertex, and the vertex becomes,

$$\Gamma^\mu = \tilde{F}_1 \gamma^\mu + \tilde{F}_2 \frac{i\sigma^{\mu\nu} q_\nu}{2M} + \tilde{F}_3 \frac{\gamma \cdot K P^\mu}{M^2}. \quad (16)$$

Then, after taking the TPE processes into considerations, the corresponding matrix elements in Eq. (11) become

$$\begin{aligned} \frac{M}{P^+} \langle N(P') \uparrow | J_{tot}^+ | N(P) \uparrow \rangle &= e(\tilde{F}_1 + \frac{1}{2}(\eta - \frac{q_L^2(\eta^2 - 1)}{4m^2\eta})\tilde{F}_3), \\ \frac{M}{P^+} \langle N(P') \uparrow | J_{tot}^+ | N(P) \downarrow \rangle &= -\frac{q_L}{2M} e(\tilde{F}_2 - \eta\tilde{F}_3), \end{aligned} \quad (17)$$

where $J_\mu^{tot} = J_{q\mu}^{1\gamma} + J_{q\mu}^{2\gamma}$. With the TPE current of the active quark and the definitions in Eq. (12) we can get some information about the TPE corrections to nucleon form factors.

4 Numerical Results and Discussions

In this work no more parameters are needed than those in the one-photon approximation [19–22]. The nucleon wave function ψ_N is quoted from Ref. [20, 22], in which the quark mass is set as $m_q = 0.22 \text{ GeV}$ and a gaussian form wave function is employed with a parameter $\beta = 0.55 \text{ GeV}$ [20].

From Eq. (17), we can easily get the TPE contributions to the electromagnetic current matrix element. After considering TPE process, there are three independent Lorentz structures in the nucleon electromagnetic vertex, so we can not separate out all the information of the TPE corrections to the nucleon form factors from the two identities in Eq. (17). As we know, the nucleon electric form factors are smaller than the magnetic form factors, especially for the neutron, so one can suppose TPE corrections to G_E^N is zero [27], namely $\Delta G_E = 0$. With this assumption, we can estimate the TPE effect on the nucleon form factors. In our calculations, we define $Y_{2\gamma}^D = \nu \tilde{F}_3 / M^2 G_D$ instead of $Y_{2\gamma} = \nu \tilde{F}_3 / M^2 G_M$ with G_D being the form factor in dipole form. In this way, we can represent the TPE corrections without considering the results under one-photon approximation. In order to make our calculations to be comparable to the experimental data, we have to consider the IR divergent part in \tilde{f}_1 , i.e. \tilde{f}_1^{soft} , separately. Similar tricks can be done as those in parton model [16, 17]. The soft parts are evaluated in a simple hadronic model with the nucleon as the intermediate state, while the contributions of the hard parts, including \tilde{f}_1^{hard} , \tilde{f}_2 and \tilde{f}_3 , are estimate from the matrix elements as shown in the left hand of Eq. (17).

In Fig. 3, we show our results for the hard part of TPE corrections to nucleon form factors at $Q^2 = 1 \text{ GeV}^2$ with the assumption $\Delta G_E = 0$. The right (left) panel is the results about the proton (neutron). For the TPE corrections to the nucleon form factors, one can find, their magnitudes are very small, but these corrections are strongly dependent on the photon polarization parameter ϵ . These features are similar to the conclusions drawn from the calculations in parton model [16, 17] and simple hadronic model [28]. Since in present work we can not separate all the TPE corrections to nucleon form factors and moreover, as we mentioned in section 2, the soft part separated from \tilde{f}_1 is not the same as

those in parton model, then in baryon level, our results about the hard part of TPE corrections to form factors are not exactly comparable to those obtained in the literatures [16, 17, 28].

After considering TPE corrections, the total unpolarized differential cross section is

$$\frac{d\sigma^t}{d\Omega} \equiv \frac{d\sigma^{1\gamma}}{d\Omega}(1 + \delta^{2\gamma}) = \frac{d\sigma^{1\gamma}}{d\Omega} + \left(\frac{d\sigma_{soft}^{2\gamma}}{d\Omega} - \frac{d\sigma_{MT}^{2\gamma}}{d\Omega} \right) + \frac{d\sigma_{hard}^{2\gamma}}{d\Omega}. \quad (18)$$

The subscript 'soft' and 'hard' denote the soft part and the hard part of TPE corrections respectively, while 'MT' means the MT corrections which have been included in the experimental data. As we have mentioned above, the soft part of the TPE corrections is evaluated in simple hadronic model, while the hard part can be evaluated as

$$\frac{d\sigma_{hard}^{2\gamma}}{d\Omega} = 2G_M^{1\gamma} Re \left[\Delta G_M + \epsilon \frac{\nu}{M^2} \tilde{F}_3 \right] + 2\frac{\epsilon}{\tau} G_E^{1\gamma} Re \left[\Delta G_E + G_D Y_{2\gamma}^D \right]. \quad (19)$$

With the assumption $\Delta G_E = 0$ and the results of TPE corrections to other form factors we can get the hard part corrections to the cross section. Here the form factors under one-photon approximation $G_{E,M}^{1\gamma}$ are taken from the Rosenbluth experimental data [29–31].

The TPE corrections to nucleon unpolarized differential cross sections are presente in Fig. 4. The right (left) panel shows the TPE corrections to proton (neutron) differential cross sections. For proton case, the TPE corrections are about 1.3% at $Q^2 = 1 \text{ GeV}^2$ and nearly 2% at $Q^2 = 3 \text{ GeV}^2$. That means, with Q^2 increasing, the TPE correction increases too, which is consistent with the conclusion drawn from parton model and simple hadronic model. However, in the simple hadronic model, when only considering nucleon as the intermediate state [28], the TPE corrections to differential cross sections are about 2% and 4% at $Q^2 = 1 \text{ GeV}^2$ and $Q^2 = 3 \text{ GeV}^2$ separately. While including $\Delta(1232)$ as well as the nucleon in the calculations, the TPE corrections to proton differential cross sections are about 1.8% and 2.8% at above two momentum transfer points. Then we can conclude that our results about TPE contributions to proton differential cross sections are comparable to those in simple hadronic model.

For neutron, our results are rather small, about 0.3% and 0.2% at $Q^2 = 1 \text{ GeV}^2$ and $Q^2 = 3 \text{ GeV}^2$ separately, which are far less than 0.8% and 1.5% in simple hadronic model [28]. In Ref. [28], only nucleon is considered as the intermediate state. When more nucleon resonances, such as $\Delta(1232)$, are included,

the TPE corrections to neutron differential cross sections are also supposed to be weaken as the case of proton [14]. It should be reiterated that in our present work we can not separate out all the TPE corrections to form factors exactly, and therefore, we can just give a rough estimate about corrections to the differential cross section.

The polarized observables P_x and P_z have extra terms after considering TPE corrections. They are expressed as:

$$\begin{aligned} P_x &= -\sqrt{\frac{2\epsilon(1-\epsilon)}{\tau}} \left(\frac{d\sigma^{un}}{d\Omega} \right)^{-1} \left\{ G_E G_M + \left[G_E \Delta G_M + G_M \Delta G_E + G_M G_D Y_{2\gamma}^D \right] \right\}, \\ P_z &= \sqrt{1-\epsilon^2} \left(\frac{d\sigma^{un}}{d\Omega} \right)^{-1} \left\{ G_M^2 + 2 \left[G_M \Delta G_M + \frac{\epsilon}{1-\epsilon} G_M G_D Y_{2\gamma}^D \right] \right\}. \end{aligned} \quad (20)$$

In above expression, the terms in the square brackets are the contributions from TPE, i.e. $P_{x,z}^{2\gamma}$. After taking $\Delta G_{E,M} = 0$ and $Y_{2\gamma}^D = 0$, the above expression will be reduced to those under one-photon exchange approximation, i.e. $P_{x,z}^{1\gamma}$. As shown in Fig. 5, we give the results for TPE corrections to the polarized observables at $Q^2 = 1 \text{ GeV}^2$. The left panel shows the ratios of $P_x^{2\gamma}$ and $P_x^{1\gamma}$ in the unit of percent. For proton, the TPE correction is about 0.5%, which is close to the results in simple hadronic model [28]. But for neutron, the correction is much larger, is about 5%. In one photon approximation, $P_x^{1\gamma}$ is proportional $G_E G_M$ and G_E^n is so small that $P_x^{1\gamma}$ is rather tiny. However, the TPE contributions contain term $G_M G_D Y_{2\gamma}^D$ and G_M^n is much larger than G_E^n , then the ratio $P_x^{1\gamma}/P_x^{2\gamma}$, for the neutron may be relative large. The right panel in Fig. 5 shows the results for the TPE corrections to P_z . For proton, the correction is about 1%, which is about two times of the results in simple hadronic model [28]. For neutron, our results is about 0.5%. The TPE corrections to neutron polarized observables keep unknown in medium Q^2 region in both simple hadronic model and parton model.

To summarize, we have studied the TPE corrections in a relativistic constituent quark model for the first time. The quark mass is found to be un-neglectable in medium momentum transfer region. In present work, we separate out the TPE contributions to form factors with the assumption $\Delta G_E = 0$, and then study the TPE contributions to the differential cross sections as well as to the polarized observables. It is found that the TPE corrections to electron-proton scattering differential cross sections are rather

small in magnitude but with strong ϵ dependence, and the TPE corrections become important at high Q^2 region. These conclusions are consistent with those drawn from parton model and simple hadronic model. However, For neutron, our results are much smaller than the results in hadronic model. This can be interpreted as not exactly extracting the TPE contributions to the form factors in our calculations as well as not including more nucleon resonances in the hadronic model. For polarized observables, the results for proton in present work are a little larger, but still comparable to those in hadronic model. Further more, there are some uncertainties for the results in hadronic model because it keeps unknown that how the nucleon resonances, especially $\Delta(1232)$, effect the TPE contributions to polarized observables. For the case of neutron, the TPE corrections to polarized observables in such low momentum transfer region have not been studied in previous literatures.

In our calculations, we also try to suppose the TPE contributions to other form factors to be zero and then study TPE contributions to electromagnetic physical observables. We can get similar results for proton, but different results for neutron due to the electric form factor of neutron is far less than the magnetic form factor. In principle, the TPE corrections can be separated by Eq. (17) together with the matrix element of other component, such as the matrix element of J^y . Unfortunately, for the contributions of zero mode [32], those separation may be much complicate and we believe that it can be evaluated in a di-quark model with light-cone formulism. This work is under process.

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Fig. 1: The Two-photon exchange process in quark level

Fig. 2: Comparisons of the real part of $\tilde{f}_i, \{i = 1, 2, 3\}$ with massive and massless quark. Here $\epsilon_q = [(s' - u')^2 + t'(4m_q - t')]/[(s' - u')^2 - t'(4m_q - t')]$. The solid curves are the results with massive quark and the dashed curves are those with massless quark.

Fig. 3: Hard part of two-photon exchange contributions to nucleon form factors. The solid lines are the corrections to magnetic form factors and the dashed lines are those for $Y_D^{2\gamma}$.

Fig. 4: Two-photon exchange contributions to unpolarized differential cross sections. The solid lines are the results at $Q^2 = 1 \text{ GeV}^2$, while the dashed lines are those at $Q^2 = 3 \text{ GeV}^2$.

Fig. 5: Two-photon exchange contributions to polarized observables at $Q^2 = 1 \text{ GeV}^2$. The solid lines are results for proton and the dashed lines are those for neutron.

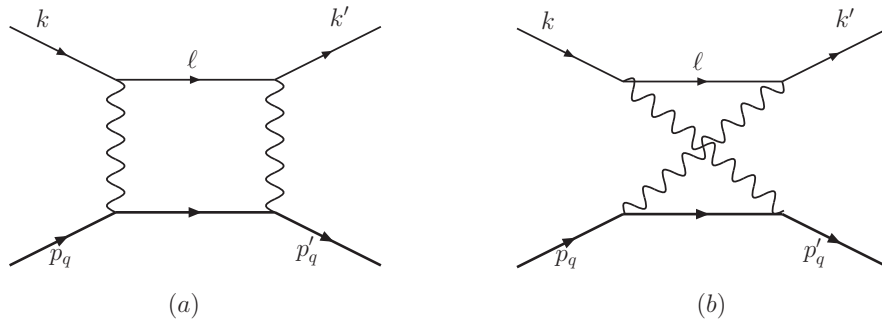


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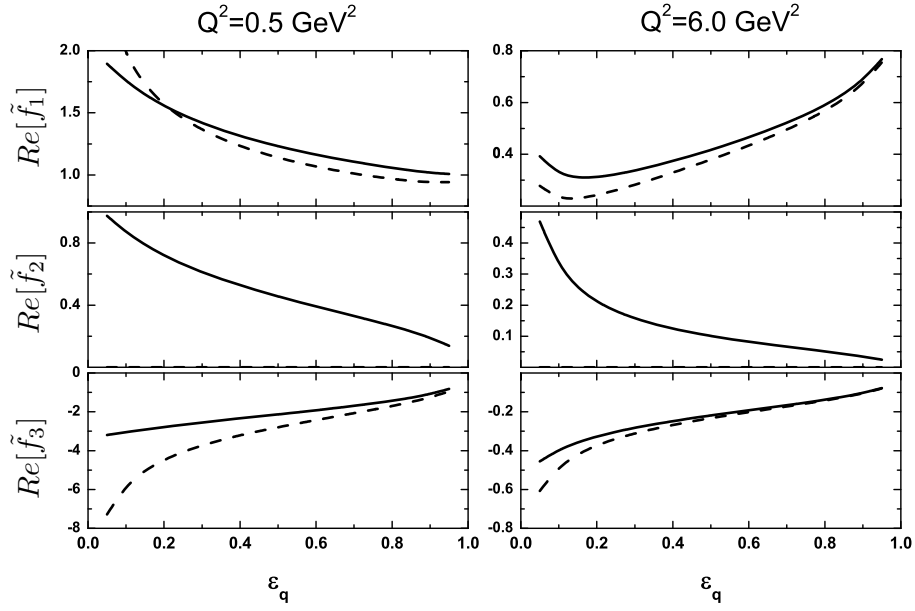


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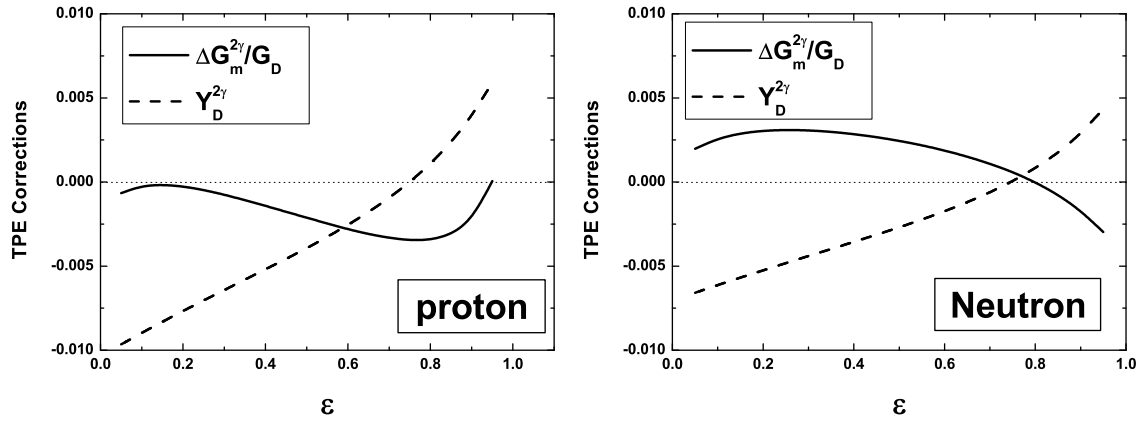


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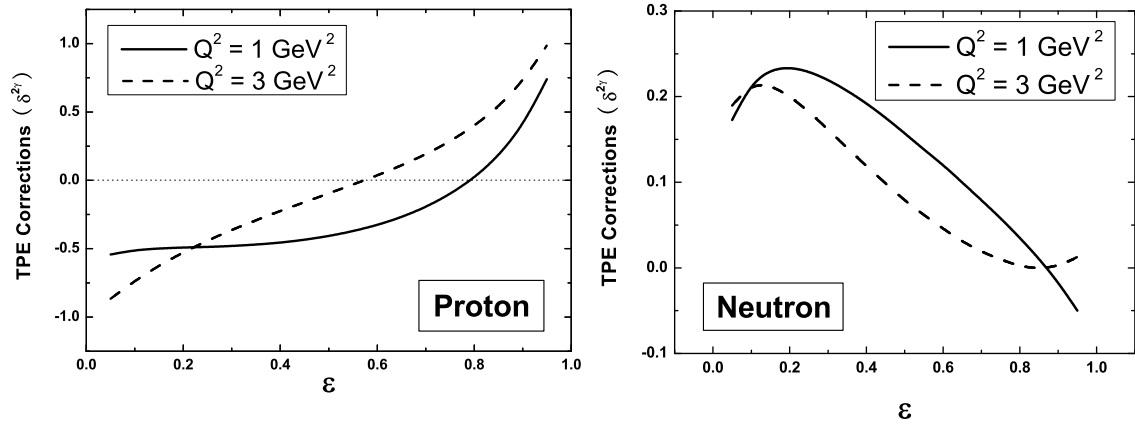


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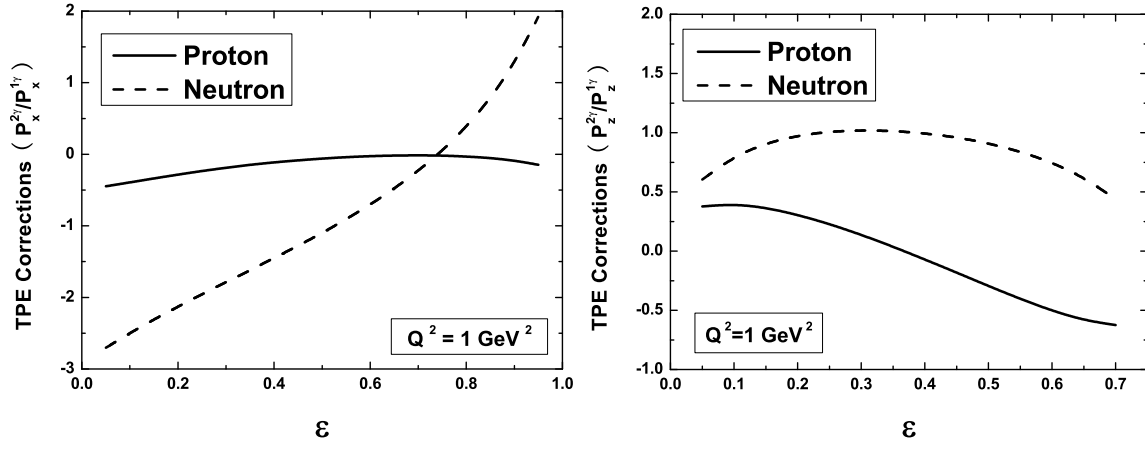


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